

Topologically protected quantum gates for computation with non-Abelian anyons in the Pfaffian quantum Hall state

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We extend the topological quantum computation scheme using the Pfaffian quantum Hall state, which has been recently proposed by Das Sarma et al., in a way that might potentially allow for the topologically protected construction of a universal set of quantum gates. We construct, for the first time, a topologically protected Controlled-NOT gate which is entirely based on quasihole braidings of Pfaffian qubits. All single-qubit gates, except for the $\pi/8$ gate, are also explicitly implemented by quasihole braidings. Instead of the $\pi/8$ gate we try to construct a topologically protected Toffoli gate, in terms of the Controlled-phase gate and CNOT or by a braid-group based Controlled-Controlled-Z precursor. We also give a topologically protected realization of the Bravyi–Kitaev two-qubit gate g_3 .

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I. INTRODUCTION

One of the spectacular features of the two-dimensional strongly correlated electron systems is that they might possess quasiparticle excitations obeying anyonic exchange statistics¹. Even more astonishing, the result of some quasiparticles exchanges might not be just phases but non-trivial statistics matrices and such quasiparticles are called non-Abelian. The most prominent non-Abelian candidate is the fractional quantum Hall (FQH) state, which is now routinely observed at filling factor $\nu = 5/2$ in ultra high-mobility samples^{2,3}. This expectation is based on convincing analytical and numerical evidence^{4,5,6,7} that this FQH state is most likely in the universality class of the Pfaffian state constructed by Moore and Read⁸ using correlation functions of an appropriate $1+1$ dimensional conformal field theory (CFT). While the experiments undoubtedly confirmed that FQH quasiparticle excitations in general carry fractional electric charge⁹, the effects of quantum statistics are in principle much more difficult to observe. That is why even Abelian fractional statistics has been experimentally tested only recently¹⁰. However, it turns out that the non-Abelian excitations might be much easier to observe despite their more complicated structure. Convincing proposals have been made for the detection of the non-Abelian statistics of the quasiparticles in the $\nu = 5/2$ FQH state^{11,12,13} and in the $\nu = 12/5$ FQH state (anticipated to be the $k = 3$ parafermion Hall state)^{14,15}.

In addition to its fundamental significance, the non-Abelian quantum statistics might become of practical importance in another field of quantum theory, the quantum computation¹⁶, which has been developing very fast in the recent years. Although the ideas behind quantum information processing are simply based on the well established fundamental postulates of the quantum theory, its exponentially growing computational power could not have been used in practice so far due to the unavoidable obstacles caused by decoherence as a result of interaction of the qubits with their environment. Even the remarkable breakthrough based on quantum error correction algorithms¹⁶ could not help creating a quantum computer with more than a few qubits. Recently

on this background emerged the brilliant idea of *topological quantum computation* (TQC)^{17,18,19}. Because the interactions leading to noise and decoherence are presumably local we can try to avoid them by encoding quantum information non-locally, using some global e.g., topological characteristics of the system. This started to be called *topological protection of qubit operations*—quantum information is inaccessible to local interactions, because they cannot distinguish between the computational basis states and hence cannot lead to decoherence^{17,19,20}. That is why topological gates are believed to be exact operations, which might potentially allow to construct a truly scalable fault-tolerant quantum-computation platform.

The FQH liquid is a perfect candidate for TQC because it possesses a number of topological properties which are universal, i.e., robust against the variations of the interactions details. One could in principle use the braid matrices representing the exchanges of FQH quasiparticles to implement arbitrary unitary transformations^{17,18,21}. However, because the single-qubit space is two-dimensional we need some degeneracy in order to implement the TQC scheme in terms of FQH quasiparticles, i.e., we need degenerate spaces of quasiparticle correlation functions with dimension at least 2. It is well known that the Abelian FQH quasiparticles have degenerate spaces on non-trivial manifolds such as torus. Unfortunately, these constructions are not appropriate for planar systems such as the FQH liquids though some diagonal two-qubit gates can be realized with abelian FQH anyons as in Ref. 22. On the other hand, the non-Abelian FQH quasiparticles by definition have degenerate spaces even in planar geometry and are therefore better suited for TQC. Another virtue of the TQC scheme is its expected scalability—the solution of the single qubit operations problem might turn out to be the solution in general. The only residual source of noise is due to thermally activated quasiparticle–quasihole pairs which might execute unwanted braids. Fortunately, these processes are exponentially suppressed at low temperature by the energy gap, which leads to astronomical precision of quantum information encoding.

One significant step forward in the field of TQC has been done recently: in a beautiful paper Das Sarma et al.^{19,20} pro-

posed to use the expected non-Abelian statistics of the quasiparticles in the Pfaffian FQH state to construct an elementary qubit and execute a logical NOT gate on it that could serve as a base for TQC. The construction of the NOT gate is a fairly important issue in the field of quantum computation because it underlies the qubit initialization procedure as well as the construction of the single-qubit gates and the Controlled-NOT (CNOT) gate.

The FQH state at $\nu = 5/2$ has one serious advantage compared to the other candidates for TQC—it is the most stable state, i.e., the one with the highest bulk energy gap, among all FQH states in which non-Abelian quasiparticle statistics is expected to be realized. On the other hand, one big disadvantage of this state is that the quasiparticles braiding matrices cannot be used alone for universal quantum computation because the braid group representation over the Ising model correlation functions is finite^{19,23,24}. This has to be compared with the FQH state observed at $\nu = 12/5$ ^{2,3}, whose braid matrices are expected to be universal, but whose energy gap is an order of magnitude lower than that of the $\nu = 5/2$ one. The motivation for this paper is to demonstrate that it might be possible to find a complete set of topologically protected quantum gates in the Pfaffian state, though not all of them would be realizable simply in terms of braidings. More precisely, we explicitly construct by braiding the single-qubit Hadamard gate H , phase gate S , and the two-qubit CNOT gate, which generate a Clifford group playing a central role in the error correction codes²⁵. In addition we propose a candidate for a topologically protected three-qubit Toffoli gate realized in terms of the two-qubit Controlled- S gate and CNOT or by a Controlled-Controlled- Z gate precursor realized by braiding. This combination of gates is known to be sufficient for universal quantum computation¹⁶.

II. ONE-QUBIT GATES FOR PFAFFIAN QUBITS

The Pfaffian qubit is constructed in terms of the wave functions for the excitations containing 4 quasiholes^{19,20}. For fixed positions of the quasiholes there are two independent functions forming the computational basis, which can be conveniently written as²⁶

$$|0\rangle, |1\rangle \Leftrightarrow \Psi_{4\text{qh}}^{(0,1)} = \frac{(\eta_{13}\eta_{24})^{\frac{1}{4}}}{\sqrt{1 \pm \sqrt{x}}} (\Psi_{(13)(24)} \pm \sqrt{x} \Psi_{(14)(23)}) \quad (1)$$

where η_1, \dots, η_4 are the quasiholes positions, x is a CFT invariant crossratio²⁷

$$x = \frac{\eta_{14}\eta_{23}}{\eta_{13}\eta_{24}}, \quad \text{and} \quad \eta_{ab} = \eta_a - \eta_b. \quad (2)$$

The explicit form of the functions $\Psi_{(ab)(cd)}$, which are computed as Pfaffians, will not be needed here, we shall only use that they are single-valued in the positions of the quasiholes^{26,28}. A convenient way to fix the positions of the quasiholes is to localize them on antidots¹⁹. The readout of the qubit state could be efficiently done by measuring the interference pattern of the longitudinal conductance in an electronic Mach-Zehnder interferometer^{19,20,29}.

The basic idea of the TQC scheme of Ref. 19 is that the single-qubit gates can be realized in terms of the two-dimensional exchange matrices of 4 quasiholes, defined here in the basis (1). In order to obtain the exchange matrix $R_{a,a+1}^{(4)}$ we first exchange the coordinates $\eta_a \leftrightarrow \eta_{a+1}$ in counterclockwise direction, which changes the crossratio (2) in a tractable way²³, and then use the analytic properties of the wave functions (1) to extract $R_{a,a+1}^{(4)}$. Thus we find the elementary braid matrices²³ in the basis (1)

$$R_{12}^{(4)} = R_{34}^{(4)} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \quad R_{23}^{(4)} = \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}, \quad (3)$$

which together with their inverses, generate the entire representation of the braid group \mathcal{B}_4 . The generators B_i , $i = 1, \dots, n-1$, of the braid group \mathcal{B}_n satisfy in general the Artin relations³⁰

$$B_i B_j = B_j B_i, \quad \text{for } |i-j| \geq 2 \\ B_i B_{i+1} B_i = B_{i+1} B_i B_{i+1}, \quad \text{with } B_i = R_{i,i+1} \in \mathcal{B}_n. \quad (4)$$

In our case of planar geometry the exchange matrices $B_i = R_{i,i+1}^{(n)}$ should satisfy one more relation³⁰

$$B_1 B_2 \cdots B_{n-2} B_{n-1}^2 B_{n-2} \cdots B_2 B_1 = \mathbb{I},$$

as is appropriate for the representation of the braid group on the sphere. Because this is satisfied in the Pfaffian TQC scheme only up to a phase factor, the representations of the braid groups are generically projective³⁰.

In other words, everything which can be obtained as a result of quasiparticle exchanges could be expressed in terms of the elementary exchange matrices (3). For example, skipping throughout overall phases, the Hadamard gate¹⁶ can be executed by 3 elementary braids (skipping the superscript “(4)” of R)

$$H \simeq R_{13} R_{12}^2 = R_{12}^{-1} R_{23} R_{12}^{-1} \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (5)$$

and its braid diagram in terms of the elementary exchanges is shown on Fig. 1. Similarly, the Pauli X gate, first implemented

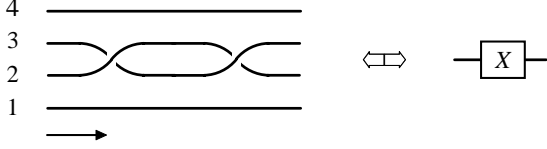
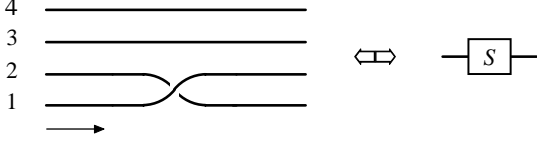


FIG. 1: Braiding diagram for the Hadamard gate (5) and its symbol (on the right) in standard quantum-computation notation.

in Ref. 19, and the phase gate¹⁶ S are expressed as

$$X = R_{23}^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad S = R_{12} = R_{34} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

and their braid diagrams are shown on Figs. 2 and 3, re-

FIG. 2: Braiding diagram for the Pauli X gate.FIG. 3: Braiding diagram for the phase gate S

spectively. Notice that $S^2 = Z$, where Z is Pauli Z gate, as it should be¹⁶. The last remaining single-qubit gate, the $\pi/8$ gate $T = \text{diag}(1, e^{i\pi/4})$, which is necessary for approximating arbitrary single-qubit gate, cannot be realized by exact braidings because $\det T = e^{i\pi/4}$, while $\det R_{a, a+1}^{(4)} = i$. Instead, we will try to construct later the Toffoli gate¹⁶.

III. TWO-QUBIT GATES: CONTROLLED-NOT

It is natural to use 6 quasiholes, as shown on Fig. 4, to construct two-qubit gates, because the 6-quasiholes wave functions form a 4-dimensional space (in general the dimension of the space of wave functions with $2n$ quasiholes with fixed positions²⁶ is 2^{n-1}). For the selected class of the controlled two-qubit operations¹⁶ the first two quasiholes determine the state of the control qubit while the last two form the target qubit. Just like with 4 quasiholes the state of a qubit is

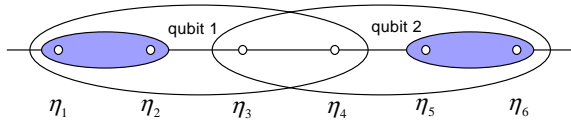


FIG. 4: Two qubits, realized by quasiholes with coordinates $(\eta_1, \eta_2, \eta_3, \eta_4)$ and $(\eta_3, \eta_4, \eta_5, \eta_6)$. The state of qubit 1 is determined by the quasiholes with positions (η_1, η_2) , while the state of qubit 2 by (η_5, η_6) and that is why these two groups are shaded.

$|0\rangle \leftrightarrow \sigma_+ \sigma_+$ if the two quasiholes fuse to the unit operator, or $|1\rangle \leftrightarrow \sigma_+ \sigma_-$ if they fuse to form a Majorana fermion¹⁹. Here we use that the neutral part of the quasihole is represented by the chiral spin field σ_{\pm} of the Ising model, with CFT dimension $1/16$, and the subscript \pm denotes its fermion parity^{23,28}. Thus we may define the two-qubit basis by

$$\begin{aligned} |00\rangle &\equiv \langle \sigma_+ \sigma_+ \sigma_+ \sigma_+ \sigma_+ \sigma_+ \rangle, & |01\rangle &\equiv \langle \sigma_+ \sigma_+ \sigma_+ \sigma_- \sigma_+ \sigma_- \rangle \\ |10\rangle &\equiv \langle \sigma_+ \sigma_- \sigma_+ \sigma_- \sigma_+ \sigma_+ \rangle, & |11\rangle &\equiv \langle \sigma_+ \sigma_- \sigma_+ \sigma_+ \sigma_+ \sigma_- \rangle. \end{aligned} \quad (6)$$

which is convenient since if we fuse the first two quasiholes this would project to the second qubit, while if we fuse the last two quasiholes we project to the first qubit²³, i.e.,

$$|\alpha\beta\rangle_{\eta_1 \rightarrow \eta_2} \rightarrow |\beta\rangle, \quad |\alpha\beta\rangle_{\eta_5 \rightarrow \eta_6} \rightarrow |\alpha\rangle. \quad (7)$$

Then the state of the third and the fourth quasiholes are fixed by the conservation of the fermion parity, i.e., if e_i is the parity of σ_{e_i} then $e_3 e_4 = e_1 e_2 e_5 e_6$ ²³. Thus we have only 4 independent states in the space of 6-quasiholes with fixed positions, which correspond to Eq. (6).

Using the two-qubit states definition (6) and the projection (7) it is easy to find the exchange matrices for 6 quasiholes from those for 4 quasiholes. For example, to obtain R_{12} , we may first fuse the last two quasiholes $\eta_5 \rightarrow \eta_6$ and then identify this braid matrix with the tensor product $R_{12}^{(6)} = R_{12}^{(4)} \otimes \mathbb{I}_2$, where \mathbb{I}_2 is the two-dimensional unit matrix and the superscript of R shows the number of quasiholes. Thus the elementary 6-quasiholes braid matrices are²³

$$R_{12}^{(6)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{pmatrix}, \quad R_{23}^{(6)} = \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -i & 0 \\ 0 & 1 & 0 & -i \\ -i & 0 & 1 & 0 \\ 0 & -i & 0 & 1 \end{pmatrix} \quad (8)$$

$$R_{34}^{(6)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad R_{45}^{(6)} = \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}} \begin{pmatrix} 1 & -i & 0 & 0 \\ -i & 1 & 0 & 0 \\ 0 & 0 & 1 & -i \\ 0 & 0 & -i & 1 \end{pmatrix} \quad (9)$$

$$R_{56}^{(6)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix}. \quad (10)$$

It is not difficult to check that the exchange matrices (8), (9) and (10) satisfy the Artin relations (4) for the braid group \mathcal{B}_6 .

One of the main advantages of our two-qubit construction (6), shown on Fig. 4, is that it is straight forward to execute various two-qubit gates by braiding. For example, using the explicit braid matrices from Eqs. (8), (9) and (10), it is easily verified that the CNOT gate can be implemented, in the basis (6), by

$$\text{CNOT} = R_{34}^{-1} R_{45} R_{34} R_{12} R_{56} R_{45} R_{34}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad (11)$$

whose braid diagram is shown on Fig. 5. An equivalent realization would be

$$\text{CNOT} = R_{56} R_{45} R_{56}^{-1} R_{34}^{-1} R_{12} R_{45} R_{56}$$

giving precisely the same result as in Eq. (11). It is quite remarkable that the CNOT gate can be implemented entirely in terms of 6-quasiholes braidings which certainly guarantees its exactness and topological protection. The best achievement in this direction so far has been the proposed realization²⁰ of a Controlled- Z gate³¹ precursor by taking (η_5, η_6) around η_1 , which corresponds in our approach to $R_{15}^2 R_{16}^2 \simeq R_{56}^2$, i.e.,

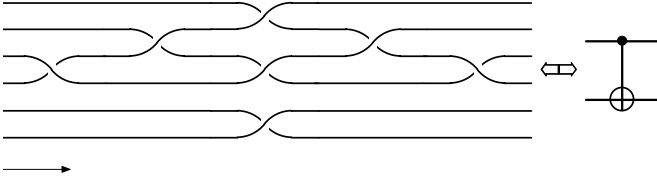


FIG. 5: The braid diagram for the Controlled-NOT gate executed by 7 elementary 6-quasiparticle braids corresponding to Eq. (11).

expressed in terms of elementary braids (skipping the superscript “(6)” of R)

$$\begin{aligned}\widetilde{CZ} &= R_{12}^{-1} R_{23}^{-1} R_{34}^{-1} R_{45} R_{56}^2 R_{45} R_{34} R_{23} R_{12} \\ &= \text{diag}(1, -1, 1, -1).\end{aligned}$$

However, this must be supplemented by the Bravyi–Kitaev procedure²⁰ in which the quasiholes with positions η_1 and η_2 are split only if their qubit is in the state $|1\rangle$, thus removing the minus sign on the second row.

While not necessary for our TQC scheme, we give below for reference another important two-qubit gate, the g_3 gate of Bravyi–Kitaev²⁰, in terms of the 6-quasihole exchange matrix (skipping again the superscript “(6)” of R)

$$\begin{aligned}R_{16} &= R_{12}^{-1} R_{23}^{-1} R_{34}^{-1} R_{45}^{-1} R_{56} R_{45} R_{34} R_{23} R_{12} \\ &\simeq \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & -i \\ 0 & 1 & -i & 0 \\ 0 & -i & 1 & 0 \\ -i & 0 & 0 & 1 \end{pmatrix} = g_3.\end{aligned}\quad (12)$$

IV. THREE-QUBIT GATES

In order to make our scheme universal we may try, instead of using T , to construct the Toffoli gate Controlled-Controlled-NOT (CCNOT) in terms of the Controlled-Controlled-Z gate (CCZ) and the Hadamard gate acting on the target qubit

$$\text{CCNOT} = H_3 \text{CCZ} H_3, \quad \text{with} \quad H_3 = \mathbb{I}_2 \otimes \mathbb{I}_2 \otimes H, \quad (13)$$

where $\text{CCZ} = \text{diag}(1, 1, 1, 1, 1, 1, -1)$. When the third qubit is defined by 2 more quasiholes at η_7 and η_8 we can express (using the fusion rules of the non-Abelian quasiholes) the exchange matrices for 8 quasiholes recursively in terms of those for 6 quasiholes as follows²³:

$$\begin{aligned}R_{12}^{(8)} &= R_{12}^{(6)} \otimes \mathbb{I}_2, & R_{23}^{(8)} &= R_{23}^{(6)} \otimes \mathbb{I}_2, \\ R_{34}^{(8)} &= \text{diag}(1, i, i, 1, i, 1, 1, i), \\ R_{45}^{(8)} &= R_{45}^{(6)} \otimes \mathbb{I}_2, & R_{56}^{(8)} &= R_{56}^{(6)} \otimes \mathbb{I}_2\end{aligned}\quad (14)$$

$$R_{67}^{(8)} = \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 1 & -i & 0 & 0 & 0 & 0 & 0 \\ 0 & -i & 1 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 & 1 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & -i & 1 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 & 0 & 1 \end{pmatrix}, \quad (15)$$

and

$$R_{78}^{(8)} = \mathbb{I}_2 \otimes R_{56}^{(6)}. \quad (16)$$

The direct check shows that the exchange matrices (14), (15) and (16) satisfy the Artin relations (4) for the braid group \mathcal{B}_8 .

Now we can explicitly construct the three-qubit Hadamard gates in terms of the exchange matrices for 8 quasiholes (skipping the superscript “(8)” of R), e.g., the Hadamard gate acting on the first qubit is

$$H_1 = H \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \simeq R_{12}^{-1} R_{23}^{-1} R_{12}^{-1}$$

and that acting on the second qubit is

$$H_2 = \mathbb{I}_2 \otimes H \otimes \mathbb{I}_2 \simeq R_{56}^{-1} R_{45}^{-1} R_{56}^{-1}.$$

Instead of the Hadamard gate H_3 acting on the third qubit, which is more difficult to construct, we can use in Eq. (13)

$$\tilde{H}_3 = R_{78} R_{45} R_{56} R_{67}^{-1} R_{56} R_{45} R_{78} \simeq \mathbb{I}_2 \otimes \mathbb{I}_2 \otimes H, \quad (17)$$

that still does the job of expressing CCNOT in terms of CCZ, while differing slightly from H_3 . Note that, due to a specific feature built into the braid group representation, it may not be always possible to represent exactly the single- and two-qubit gates in the three-qubit basis²³.

Next, because $Z = S^2$, we could construct¹⁶ the CCZ gate with the circuit shown on Fig. 6, using the CNOT gate (11)

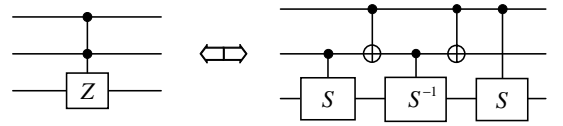


FIG. 6: The Controlled-Controlled-Z gate realized by the Controlled-S gate and CNOT (see Ref. 16 for the notation).

and the Controlled-S gate (CS) defined in the basis (6) by $\text{CS} = \text{diag}(1, 1, 1, i)$. Obviously CS cannot be expressed directly as a product of 6-quasiholes exchange matrices because $\det \text{CS} = i$, while $\det(R_{a, a+1}^{(6)}) = -1$. We can try to realize the CS gate using as a braid-group based precursor the braid matrix $R_{56}^{(6)}$ in Eq. (10) supplementing it by the Bravyi–Kitaev construction to split the first qubit into two separated quasiholes at η_1 and η_2 only if it is in the state $|1\rangle$. In the standard quantum computation context, the CS gate can be realized with the help of the $\pi/8$ gate T and CNOT, as shown on Fig. 7, which demonstrates that the use of the Toffoli gate is equivalent to the use of T .

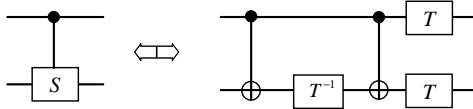


FIG. 7: The Controlled- S gate in terms of the $\pi/8$ gate T and CNOT.

The construction of the CNOT gate in Eq. (11) suggests another possibility to realize the CCZ gate using as a braid-group based precursor the diagonal matrix

$$\begin{aligned} \widetilde{\text{CCZ}} &\equiv R_{12}^{(8)} R_{34}^{(8)} R_{56}^{(8)} R_{78}^{(8)} \\ &\simeq \text{diag}(-1, 1, 1, 1, 1, 1, 1, -1) \end{aligned} \quad (18)$$

and apply again the Bravyi–Kitaev construction that the first qubit is split into two charge- $1/4$ quasiholes only if it is in the state $|1\rangle$ in order to remove the minus sign on the first row. This is precisely the same situation as with the CZ precursor of Ref. 20, where the Bravyi–Kitaev construction removing the minus sign on the second row of their gate g_2 , is believed to be realizable by tilted interferometry in planar geometry. Note that the tilted interferometry realization of the Bravyi–Kitaev construction²⁰ for Eq. (18) could eventually provide us with a topologically protected CCZ gate, hence, with a universal set of topologically protected gates that are sufficient for universal quantum computation with Pfaffian qubits.

Alternatively, to make our scheme universal, we could use the unprotected $\pi/8$ gate U_P of Ref. 20, or use the magic states of Ref. 25, such as $|H\rangle = \cos(\pi/8)|0\rangle + \sin(\pi/8)|1\rangle$, which

in principle allow to build the T gate, while being simpler to construct than the $\pi/8$ gate itself.

V. CONCLUSION

We demonstrated that it might indeed be possible to build a universal quantum computer with topologically protected quantum gates realized by quasiparticle braidings in the FQH state at $\nu = 5/2$. Our quantum computation scheme is based on the Hadamard gate H , the phase gate S , the CNOT and possibly the Toffoli gate expressed in terms the Controlled- S gate. If in addition the CCZ gate could be realized by tilted interferometry from the braid-group based precursor proposed here, then we would be able to construct the Toffoli gate and hence implement arbitrary quantum gates in a topologically protected way.

We should stress that it is not completely trivial to embed all one-qubit and two-qubit gates, realized in this paper, into systems with three or more qubits because the construction of some of those gates is based on specific properties of the generators of braid groups \mathcal{B}_4 and \mathcal{B}_6 , which are not directly generalized for \mathcal{B}_n with $n \geq 8$. Nevertheless, the embedding of the Clifford gates H , S and CNOT into a three-qubit system seems possible though that might require some more work.

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³¹ the CNOT gate can be obtained from CZ with the help of the

Hadamard gate acting on the target qubit¹⁶